Ising model spin $S = 1$ on directed Barabási-Albert networks

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Abstract: On directed Barabási-Albert networks with two and seven neighbours selected by each added site, the Ising model with spin $S = 1/2$ was seen not to show a spontaneous magnetisation. Instead, the decay time for flipping of the magnetisation followed an Arrhenius law for Metropolis and Glauber algorithms, but for Wolff cluster flipping the magnetisation decayed exponentially with time. On these networks the Ising model spin $S = 1$ is now studied through Monte Carlo simulations. However, in this model, the order-disorder phase transition is well defined in this system. We have obtained a first-order phase transition for values of connectivity $m = 2$ and $m = 7$ of the directed Barabási-Albert network.

Keywords:Monte Carlo simulation, Ising , networks, desorden.

Introduction

Sumour and Shabat [1, 2] investigated Ising models with spin $S = 1/2$ on directed Barabási-Albert networks [3] with the usual Glauber dynamics. No spontaneous magnetisation was found (and we now confirmed this effect), in contrast to the case of undirected Barabási-Albert networks [4, 5, 6] where a spontaneous magnetisation was found lower a critical temperature which increases logarithmically with system size. More recently, Lima and Stauffer [7] simulated directed square, cubic and hypercubic lattices in two to five dimensions with heat bath dynamics in order to separate the network effects form the effects of directedness. They also compared different spin flip algorithms, including cluster flips [8], for Ising-Barabási-Albert networks. They found a freezing-in of the magnetisation similar to [1, 2], following an Arrhenius law at least in low dimensions. This lack of a spontaneous magnetisation (in the usual sense) is consistent with the fact that if on a directed lattice a spin S_j influences spin S_i, then spin S_i in turn does not influence S_j, and there may be no well-defined total energy. Thus, they show that for the same scale-free networks, different algorithms give different results. Now we study the Ising model for spin $S = 1$ on directed Barabási-Albert network and different from the Ising model for spin $S = 1/2$, the order-disorder phase transition of order parameter is well defined in this system. We have obtained a first-order

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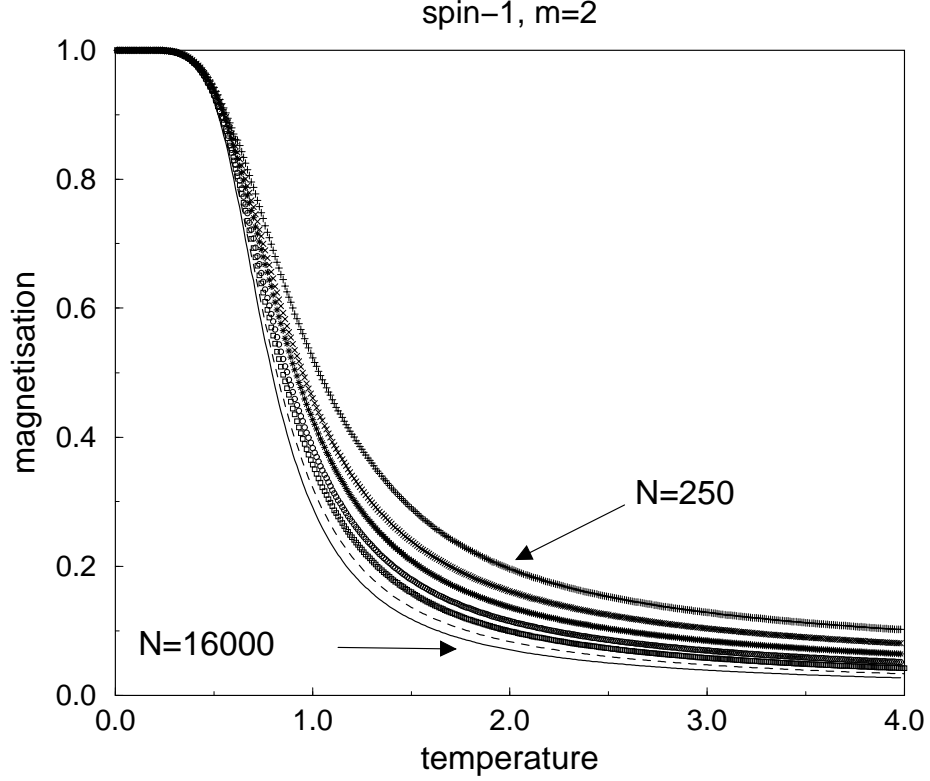phase transition for values of connectivity $m = 2$ and $m = 7$ of the directed Barabási-Albert network.

Figure 1: Plot of spontaneous magnetization versus temperature for various network sizes.

Model and Simulation

We consider the Ising model with spin $S = 1$, on directed Barabási-Albert Networks, defined by a set of spins variables S taking the values -1, 0 and $+1$, situated on every site of a directed Barabási-Albert Networks with N sites.

The Ising interation energy is given by
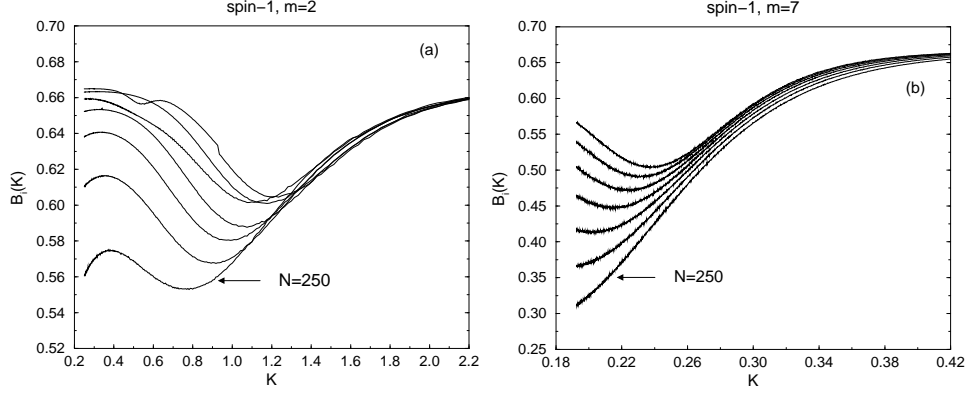
$$E = -J \sum_i \sum_k S_i S_k \qquad (1)$$

Figure 2: Plot of the Binder parameter B_i versus K for severals systems sizes (N = 250, 500, 1000, 2000, 4000, 8000 and 16000). In the Part (a) $m = 2$ and Part (b) $m = 7$.

where k-sum runs over all nearest neighbors of S_i. In this network, each new site added to the network selects m already existing sites as neighbours influencing it; the newly added spin does not influence these neighbours. To study the critical behavior of the model we define the variable $e = E/N$ and $m = \sum_{i=1}^{N} S_i/N$. ¿From variable of the energy measurements we can compute, the average energy and specific heat and energetic fourth-order parameter,

$$u(K) = [< E >]_{av}/N, \tag{2}$$

$$C(K) = K^2 N[< e^2 > - < e >^2]_{av}, \tag{3}$$

$$B_i(K) = [1 - \frac{< e^4 >}{3 < e^2 >^2}]_{av}, \tag{4}$$

where $K = J/k_B T$, with $J = 1$, and k_B is the Boltzmann constant. Similarly, we can derive from the magnetization measurements the average magnetization, the susceptibility, and the magnetic cumulants,

$$m(K) = [< |m| >]_{av}, \tag{5}$$

$$\chi(K) = KN[< m^2 > - < |m| >^2]_{av}, \tag{6}$$

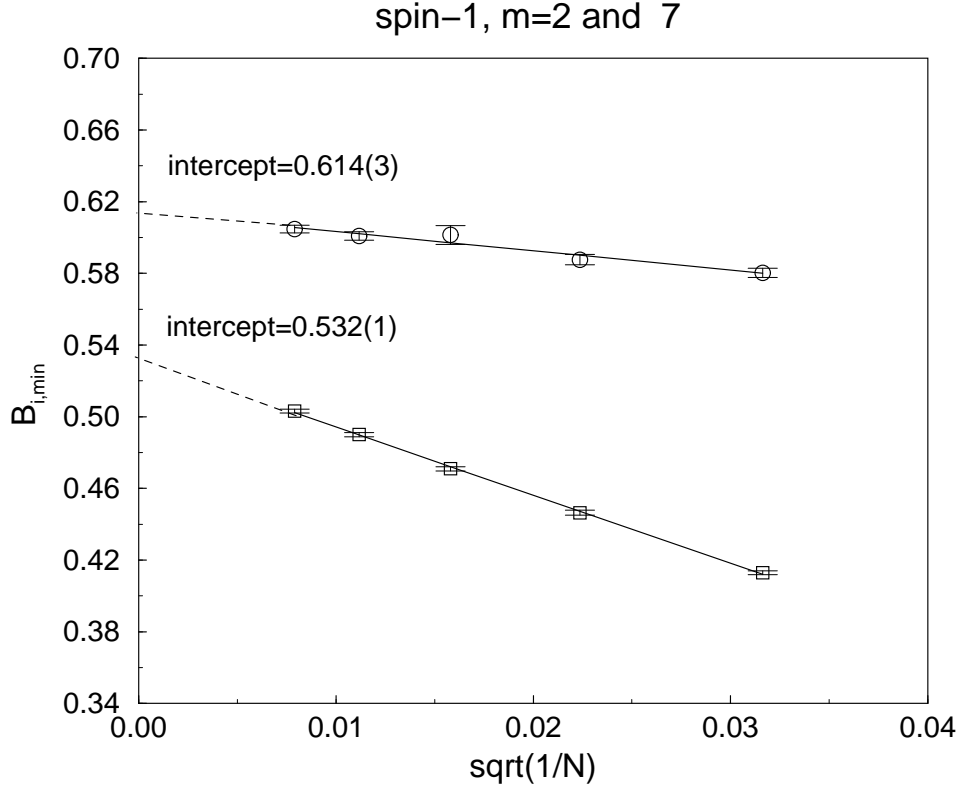$$U_4(K) = [1 - \frac{< m^4 >}{3 < |m| >^2}]_{av}. \tag{7}$$

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Figure 3: Plot of the Binder parameter B_i versus $1/N$ for $m = 2$ (circle) and $m = 7$ (square), and severals systems sizes ($N = 1000, 2000, 4000, 8000$ and 16000).

where $< ... >$ stands for a thermodynamics average and $[...]_{av}$ square brackets for a averages over the 20 realizations.

In the order to verify the order of the transition this model, we apply finite-size scaling (FSS) [9]. Initially we search for the minima of energetic fourth-order cumulant in eq. (4). This quantity gives a qualitative as well as a quantitative description of the order the transition [10]. It is known [11] that this parameter takes a minimun value $B_{i,min}$ at the effective transition temperature $T_c(N)$. One can show [12] that for a second-order transition $\lim_{N\to\infty} (2/3 - B_{i,min}) = 0$, even at T_c, while at a first-order transition the same limit measures the latent heat $|e_+ - e_-|$:
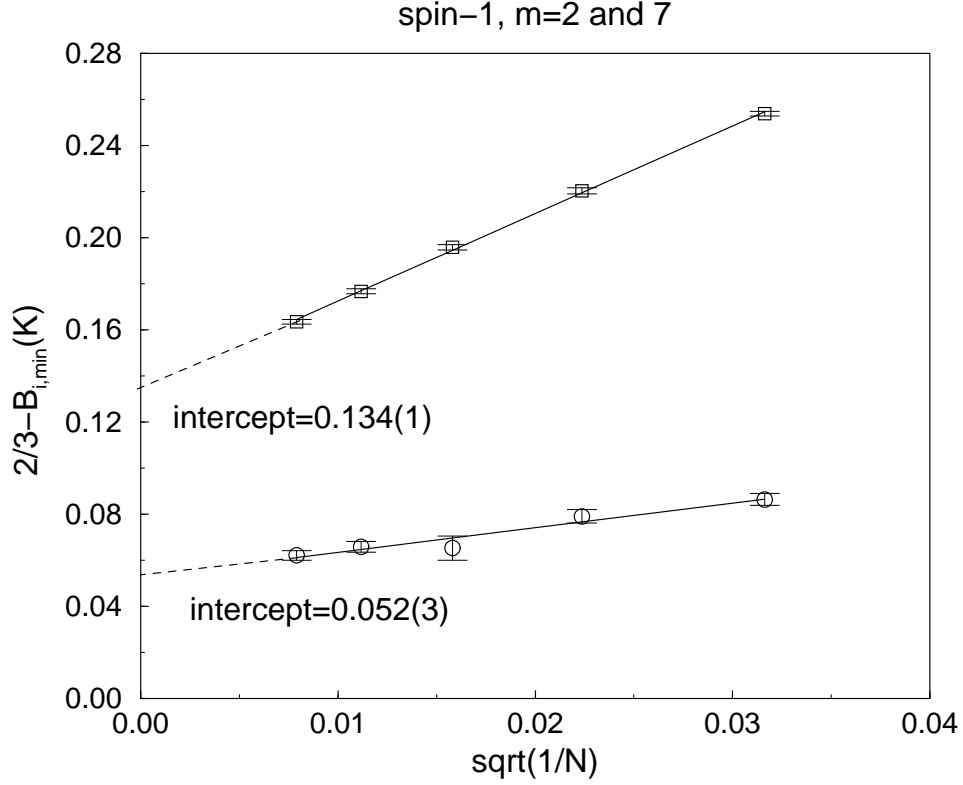
Figure 4: Plot of the Binder parameter $2/3 - B_i(K)$ versus $1/N$ for $m = 2$ (circle) and $m = 7$ (square), and severals systems sizes ($N = 1000, 2000, 4000, 8000$ and 16000).

$$\lim_{N \to \infty} (2/3 - B_{i,min}) = \frac{1}{3} \frac{(e_+ - e_-)^2 (e_+ + e_-)^2}{(e_+^2 - e_-^2)^2}. \qquad (8)$$

A more quantitative analysis can be carried out throught the FSS of the specific heat C_{max}, the susceptibility maxima χ_{max} and the minima of the Binder parameter $B_{i,min}$. If the hypothesis of a first-order phase transition is correct, we should then expect, for large systems sizes, an asymptotics FSS behavior of the form [13, 14],

$$C_{max} = a_C + b_C N + ... \qquad (9)$$
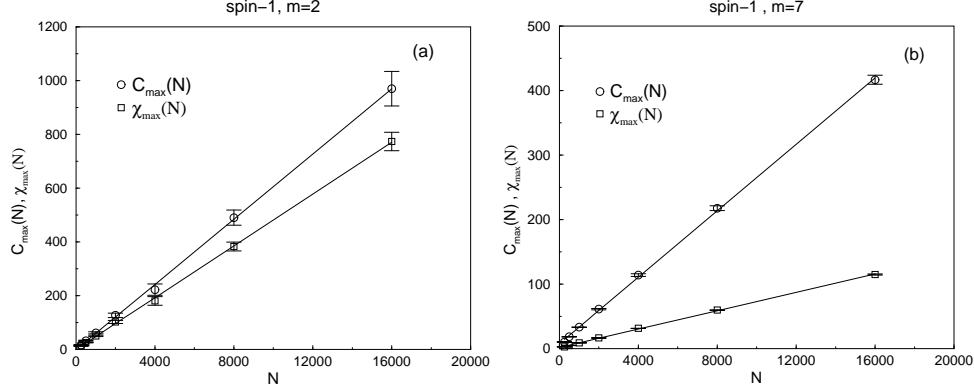
$$\chi_{max} = a_\chi + b_\chi N + ... \qquad (10)$$

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Figure 5: Plot of the specific heat C_{max} (circle) and susceptibility χ_{max} (square) versus N. In the Part (a) $m = 2$ and Part (b) $m = 7$.

$$B_{i,min} = a_{B_i} + bB_iN + ... \qquad (11)$$

We have performed Monte Carlo simulation on directed Barabási-Albert networks with values of connectivity $m = 2$ and 7. For a given m, we used systems of size $N = 250, 500, 1000, 2000, 4000, 8000$, and 16000 sites. We waited 10000 Monte Carlo steps (MCS) to make the system reach the steady state, and the time averages were estimated from the next 10000 MCS. In our simulations, one MCS is accomplished after all the N spins are updated. For all sets of parameters, we have generated 20 distinct networks, and have simulated 20 independent runs for each distinct network.

Results and Discussion
Our simulations, using the HeatBath algorithm, indicate that the model displays a first order phase transition. Fig.1 shows the overall behaviour of the spontaneous magnetisation. In Fig. 2 we show the dependence of the Binder parameter $B_i(K)$ for connectivity $m = 2$ and 7 on the inverse of temperature K and various systems size. Part (a) shows the curves for $m = 2$ of bottom to top of $N = 250$ to 16000, part (b) the same as part (a) for $m = 7$. The Binder parameter clearly goes to a value which is different from 2/3. This is a sufficient condition to characterize a first-order transition. In Fig. 3 we plot the Binder parameter B_i versus $1/N$ for $m = 2$ (circle) and $m = 7$ (square), and severals systems sizes ($N = 1000, 2000, 4000, 8000$ and 16000). We show the scaling of the Binder parameter minima, and again

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the first order phase transition is verified. The order of the transitions can be confirmed by ploting the values of $2/3 - B_{i,min}$ again versus $1/N$. For a second-order transition the curves goes to zero as we increase the system size. Here, the quantity $2/3 - B_{i,min}$ approaches a nonvanishing value in the limit of small $1/N$ as for $m = 2$ than as $m = 7$, see Fig. 4.

As decipted in Figure 5, our results for scaling of the specific heat and susceptibility are consistent with equations (9,10). Part a shows $m = 2$, and part b, $m = 7$. The same occurs with the plot the Fig. 4 for the Binder parameter minima, equation (11). In the part a for $m = 2$ and $m = 7$ part b.

Conclusion

In conclusion, we have presented a very simple equilibrium model on directed Barabási-Albert network [1, 2]. Different from the spin 1/2 Ising model, in these networks, the spin 1 Ising model presents a the first-order phase transition which occurs in model with connectivity $m = 2$ and $m = 7$ here studied. We also verific that occur a phase transition for Potts Model for $q = 3$ and $q = 8$ on directed Barabási-Albert network [15].

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